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Rituel : $\int_{-1}^{3} \frac{2}{(3x+5)^2}dx$ $n=2$ $\int \frac{u'}{u^2} = -\frac{1}{u}$

$u(x)=3x+5$ donc $u'(x)=3$

$\frac{2}{(3x+5)^2} = \frac{2}{3} \times \frac{3}{(3x+5)^2}$ donc $\int_{-1}^{3} \frac{2}{(3x+5)^2}dx = \left[\frac{2}{3} \times (-1) \times \frac{1}{3x+5}\right]_{-1}^{3} = -\frac{2}{3}\left(\frac{1}{3\times 3+5} - \frac{1}{3\times(-1)+5}\right)$

$= -\frac{2}{3}\left(\frac{1}{14} - \frac{1}{2}\right) = \frac{2}{7}$

La normale :

démonstration de $E(X)=0$ si $X \sim \mathcal{N}(0;1)$

On admet que :

$E(X) = \lim_{x \to -\infty} \int_{x}^{0} t f(t)dt + \lim_{y \to +\infty} \int_{0}^{y} t f(t)dt$. *

$\int_{x}^{0} t f(t)dt = \int_{x}^{0} t \times \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt =$

$u(t) = -t^2/2 = -\frac{1}{2}t^2$ donc $u'(t) = -\frac{1}{2} \times 2t = -t$

$\int_{x}^{0} t f(t)dt = \int_{x}^{0} -\frac{1}{\sqrt{2\pi}} \times (-t) e^{-t^2/2} dt = \left[-\frac{1}{\sqrt{2\pi}} \times \exp\left(-\frac{t^2}{2}\right)\right]_{x}^{0} = \boxed{-\frac{1}{\sqrt{2\pi}} \times \left(1 - e^{-x^2/2}\right)}$

$\lim_{x \to -\infty} -\frac{x^2}{2} = -\infty$
$\lim_{X \to -\infty} e^X = \boxed{0}$
grâce au théorème sur les limites de fonctions composées $\lim_{x \to -\infty} e^{-x^2/2} = \boxed{0}$

donc $\lim_{x \to -\infty} \int_{x}^{0} t f(t)dt = -\frac{1}{\sqrt{2\pi}}$ grâce aux règles opératoires

de même, $\int_{0}^{y} t f(t)dt = \left[-\frac{1}{\sqrt{2\pi}} \exp\left(-\frac{t^2}{2}\right)\right]_{0}^{y} = -\frac{1}{\sqrt{2\pi}}\left(e^{-y^2/2} - 1\right)$

donc $\lim_{y \to +\infty} \int_{0}^{y} t f(t)dt = \frac{1}{\sqrt{2\pi}}$

$E(X) = -\frac{1}{\sqrt{2\pi}} + \frac{1}{\sqrt{2\pi}} = 0$