# Exponentielle complexe

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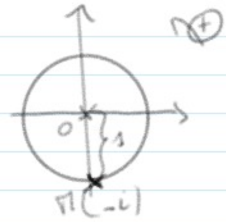
Suite de l'exercice 157 page 269

c) $z_3 = \dfrac{\sin\frac{\pi}{12} - i\cos\frac{\pi}{12}}{\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}} = \dfrac{-i\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)}{e^{i\pi/12}} = \dfrac{e^{-i\pi/2} \times e^{i\pi/12}}{e^{i\pi/12}} = e^{-i\pi/2}$




$|-i| = \sqrt{0 + (-1)^2} = 1$

**155.** On pose $z = 1 + e^{i\theta}$.

Déterminer le module ainsi qu'un argument de z, lorsque :

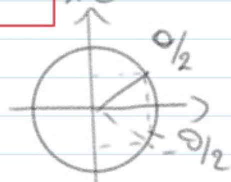
a. $\theta \in ]0\,;\pi[$ ;

b. $\theta \in ]\pi\,;2\pi[$.

$0 = \frac{\theta}{2} - \frac{\theta}{2}$ et $\theta = \frac{\theta}{2} + \frac{\theta}{2}$

$z = 1 + e^{i\theta} = e^{0} + e^{i\theta} = e^{i\left(\frac{\theta}{2} - \frac{\theta}{2}\right)} + e^{i\left(\frac{\theta}{2} + \frac{\theta}{2}\right)} = \boxed{e^{i\left(\frac{\theta}{2}\right)}} \times e^{-i\frac{\theta}{2}} + \boxed{e^{i\left(\frac{\theta}{2}\right)}} \times e^{i\frac{\theta}{2}}$

$= e^{i\theta/2} \times \underbrace{\left(e^{-i\theta/2} + e^{i\theta/2}\right)}$




$e^{-i\theta/2} = \cos\left(-\frac{\theta}{2}\right) + i\sin\left(-\frac{\theta}{2}\right) = +\cos\left(\frac{\theta}{2}\right) - i\sin\left(\frac{\theta}{2}\right)$

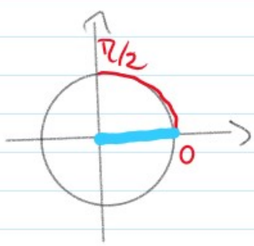
la fonction cosinus est paire
la fonction sinus est impaire

$+$

$e^{i\theta/2} = \cos\left(\frac{\theta}{2}\right) + i\sin\left(\frac{\theta}{2}\right)$.

$= \quad e^{-i\theta/2} + e^{i\theta/2} = 2\cos\left(\frac{\theta}{2}\right)$

$z = 1 + e^{i\theta} = e^{i\theta/2} \times 2\cos\left(\frac{\theta}{2}\right)$



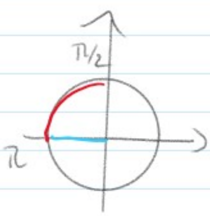

1er cas : $\theta \in ]0\,;\pi[$

$\frac{\theta}{2} \in ]0\,;\frac{\pi}{2}[$ donc $2\cos\frac{\theta}{2} > 0$

donc $|z| = 2\cos\frac{\theta}{2}$ car $2\cos\frac{\theta}{2} > 0$. et $z = 2\cos\frac{\theta}{2} \times e^{i\theta/2}$

$r > 0$

$\arg z = \frac{\theta}{2} \pmod{2\pi}$




2ième cas $\theta \in ]\pi, 2\pi[$

$\frac{\theta}{2} \in ]\frac{\pi}{2}\,;\pi[$

$\cos\frac{\theta}{2} < 0$

$\cos\frac{\theta}{2} < 0$

$$z = 1 + e^{i\theta} = \underbrace{2\cos\frac{\theta}{2}}_{<0} e^{i\theta/2} = \underbrace{\left(-2\cos\frac{\theta}{2}\right)}_{>0} \times \left(-e^{+i\theta/2}\right)$$

$$z = -2\cos\frac{\theta}{2} \times e^{i\pi} \times e^{+i\theta/2} = -2\cos\frac{\theta}{2} \times e^{i(\pi+\theta/2)}$$

$|z| = -2\cos\frac{\theta}{2}$ ⚠ $-2\cos\frac{\theta}{2} > 0$

$\arg z = \pi + \frac{\theta}{2} \pmod{2\pi}$

# Exponentielle complexe

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Démonstration des propriétés du cours :

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$+ \quad e^{-i\theta} = \cos(-\theta) + i\sin(-\theta) = \cos\theta - i\sin\theta$$

$$e^{i\theta} + e^{-i\theta} = 2\cos\theta + 0$$

donc $\boxed{\dfrac{e^{i\theta}+e^{-i\theta}}{2} = \cos\theta}$

de même, $e^{i\theta} = \cos\theta + i\sin\theta$

$$- \quad e^{-i\theta} = \cos\theta - i\sin\theta$$

$$e^{i\theta} - e^{-i\theta} = 0 + 2i\sin\theta$$

donc $\boxed{\dfrac{e^{i\theta}-e^{-i\theta}}{2i} = \sin\theta}$

$(a+b)^2$

ex : $\cos^2 x = (\cos x)^2 = \left(\dfrac{e^{ix}+e^{-ix}}{2}\right)^2 = \dfrac{\left(e^{ix}+e^{-ix}\right)^2}{4} = \dfrac{\left(e^{ix}\right)^2 + 2e^{ix}\times e^{-ix} + \left(e^{-ix}\right)^2}{4}$

$= \dfrac{e^{2ix} + 2e^{ix-ix} + e^{-2ix}}{4} = \dfrac{\boxed{e^{2ix} + e^{-2ix}} + 2}{4} = \dfrac{2\cos 2x + 2}{4} = \dfrac{1}{2}\cos 2x + \dfrac{1}{2}$

$$\frac{e^{2ix}+e^{-2ix}}{2} = \cos(2x)$$

$\sin^2 x =$