Exponentielle complexe.

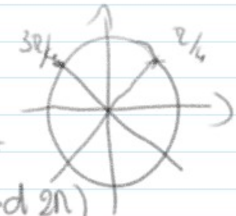
Exercice 152 page 268

152. Écrire sous forme exponentielle les nombres :
$a = -2 + 2i$ et $b = -3 - i\sqrt{3}$.
En déduire la forme exponentielle du nombre complexe ab.

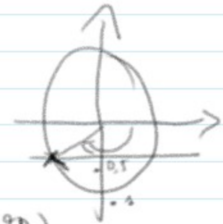
$|a| = \sqrt{(-2)^2 + 2^2} = 2\sqrt{2}$

$a = 2\sqrt{2}\left(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)$

$\begin{cases} \cos\theta = -\frac{2}{2\sqrt{2}} = -\frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{-\sqrt{2}}{2} \\ \sin\theta = \frac{2}{2\sqrt{2}} = \frac{\sqrt{2}}{2} \end{cases} \iff \theta = \frac{3\pi}{4} \pmod{2\pi}$



$a = 2\sqrt{2}\,e^{3i\pi/4}$



$|b| = \sqrt{(-3)^2 + (\sqrt{3})^2} = \sqrt{12} = 2\sqrt{3}$

$\begin{cases} \cos\theta' = -\frac{3}{2\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{-\cancel{3}\sqrt{3}}{2 \times \cancel{3}} = \frac{-\sqrt{3}}{2} \\ \sin\theta' = -\frac{\sqrt{3}}{2\sqrt{3}} = -\frac{1}{2} \end{cases} \iff \theta' = \frac{7\pi}{6} \pmod{2\pi}$

$b = 2\sqrt{3} \times \left(\boxed{\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6}}\right) = 2\sqrt{3} \times e^{7i\pi/6}$

$b = 2\sqrt{3} \times e^{7i\pi/6}$

$ab = 2\sqrt{2}\,e^{3i\pi/4} \times 2\sqrt{3}\,e^{7i\pi/6} = \underbrace{2\sqrt{2} \times 2\sqrt{3}} \times e^{3i\pi/4} \times e^{7i\pi/6} = 4\sqrt{6} \times e^{3i\pi/4 + 7i\pi/6} = 4\sqrt{6} \times e^{23i\pi/12}$

$= 4\sqrt{6} \times e^{-i\pi/12}$

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158. On donne $z = 3\sqrt{3} + 3i$.
1. Donner la forme exponentielle de z.
2. En déduire la forme exponentielle de $-z$; z^2 et $\frac{1}{z}$.

$|z| = \sqrt{(3\sqrt{3})^2 + 3^2} = \sqrt{36} = 6.$

$\begin{cases} \cos\theta = \frac{3\sqrt{3}}{6} = \frac{\sqrt{3}}{2} \\ \sin\theta = \frac{3}{6} = \frac{1}{2} \end{cases} \iff \theta = \frac{\pi}{6} \pmod{2\pi}$

$z = 6 \times e^{i\pi/6}$

2) $-z = -6e^{i\pi/6} = (-1) \times 6e^{i\pi/6} = e^{i\pi} \times 6e^{i\pi/6} = 6e^{7i\pi/6}$.

$z^2 = \left(6e^{i\pi/6}\right)^2 = 36\,e^{2i\pi/6} = 36\,e^{i\pi/3}$.

$\frac{1}{z} = \frac{1}{6e^{i\pi/6}} = \frac{1}{6} \times \frac{1}{e^{i\pi/6}} = \frac{1}{6} \times e^{-i\pi/6}$

Exercice 157 page 269.

157. Écrire les nombres suivants sous forme exponentielle :

a. $z_1 = (3 - i\sqrt{3})^4$;

b. $z_2 = \frac{1 + i\sqrt{3}}{\sqrt{3} - i}$;

c. $z_3 = \frac{\sin\frac{\pi}{12} - i\cos\frac{\pi}{12}}{\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}}$.

$z_1 = (3 - i\sqrt{3})^4$

$|3 - i\sqrt{3}| = \sqrt{3^2 + (-\sqrt{3})^2} = \sqrt{12} = 2\sqrt{3}$

$\cos\theta = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2} \quad \iff \theta = \frac{\pi}{\ } \ (2\pi)$

$$\begin{cases} \cos\theta = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2} \\ \sin\theta = -\frac{\sqrt{3}}{2\sqrt{3}} = -\frac{1}{2} \end{cases} \iff \theta = -\frac{\pi}{6} \quad (2\pi)$$

$$z_1 = \left(2\sqrt{3} \times e^{-i\pi/6}\right)^4 = \left(2\sqrt{3}\right)^4 \times \left(e^{-i\pi/6}\right)^4 \qquad ⚠ \ (a \times b)^m = a^m \times b^m$$

$$= 144\, e^{-4i\pi/6} = 144\, e^{-2i\pi/3}$$

b) $z_2 = \dfrac{1 + i\sqrt{3}}{\sqrt{3} - i}$

$|1 + i\sqrt{3}| = \sqrt{1^2 + \sqrt{3}^2} = 2$ $\qquad 1 + i\sqrt{3} = 2\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = 2e^{i\pi/3}$

$|\sqrt{3} - i| = \sqrt{\sqrt{3}^2 + (-1)^2} = 2$ $\qquad \sqrt{3} - i = 2\left(\frac{\sqrt{3}}{2} - \frac{1}{2}i\right) = 2e^{-i\pi/6}$

$$z_2 = \frac{\cancel{2}e^{i\pi/3}}{\cancel{2}e^{-i\pi/6}} = e^{i\pi/3 - (-i\pi/6)} = e^{i\pi/2} \quad (= i)$$

c) $z_3 = \dfrac{\sin\frac{\pi}{12} - i\cos\frac{\pi}{12}}{\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}} = \dfrac{-i\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)}{e^{i\pi/12}}$