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$$I = \int_0^{\pi/4} \frac{\cos 2x}{(2+3\sin 2x)^3}\,dx$$

On pose $\begin{cases} u(x) = 2 + 3\sin \boxed{2x} \\ u'(x) = 3 \times \boxed{2} \times \cos 2x = 6\cos 2x \end{cases}$

$(\sin 2x)' = 2 \times \cos 2x$

donc $\dfrac{\cos 2x}{(2+3\sin 2x)^3} = \boxed{\dfrac{1}{6}} \times \dfrac{6\cos 2x}{(2+3\sin 2x)^3}$ de la forme $\dfrac{1}{6} \times \dfrac{u'}{u^n}$ avec $n = 3$

donc $\displaystyle\int_0^{\pi/4} \frac{\cos 2x}{(2+3\sin 2x)^3}\,dx = \left[\underbrace{\frac{1}{6} \times \left(-\frac{1}{2}\right)} \times \frac{1}{(2+3\sin 2x)^2} \right]_0^{\pi/4}$

donc $I = -\dfrac{1}{12}\left(\dfrac{1}{25} - \dfrac{1}{4}\right) = \dfrac{21}{1200} = 0{,}0175$

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$$f(x) = \frac{2x+3}{(x+3)^3}$$

$u(x) = x+3$ $\quad \frac{u'}{u^3} = \frac{1}{(x+3)^3}$ $\quad \frac{2x+3}{(x+3)^3} = (2x+3) \times \frac{1}{(x+3)^3}$ n'aboutit pas.

$u'(x) = 1.$ $\quad \neq c^{st}$

1) Autre écriture de $f(x)$ Pour tout x de $]-\infty ; -3[$:

$$\frac{a}{(x+3)^2} + \frac{b}{(x+3)^3} = \frac{a}{(x+3)^2} \times \frac{x+3}{x+3} + \frac{b}{(x+3)^3} = \frac{ax + \boxed{3a+b}}{(x+3)^3} = \frac{2x + \boxed{3}}{(x+3)^3}$$

ssi $\begin{cases} a = 2 \\ 3a + b = 3 \end{cases}$ par identification ssi $\begin{cases} a = 2 \\ b = -3 \end{cases}$ donc $f(x) = \frac{2}{(x+3)^2} - \frac{3}{(x+3)^3}$

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2) $f(x) = \underbrace{\dfrac{2}{(x+3)^2}} - \underbrace{\dfrac{3}{(x+3)^3}}$

On pose : $u(x) = x+3$ donc $u'(x) = 1$ → de la forme $\dfrac{u'}{u^n}$

$f(x) = 2 \times \dfrac{1}{(x+3)^2} - 3 \times \dfrac{1}{(x+3)^3}$

Primitive de f : $F(x) = \underbrace{2 \times (-1) \times \dfrac{1}{(x+3)}} \underbrace{- 3 \times \left(-\dfrac{1}{2}\right) \times \dfrac{1}{(x+3)^2}} + C = \dfrac{-2}{(x+3)} + \dfrac{3}{2(x+3)^2} + C$

Or

$F(-4) = 0$ (cf énoncé) $\Leftrightarrow F(-4) = \dfrac{-2}{-1} + \dfrac{3}{2 \times 1} + C = 0 \Leftrightarrow \dfrac{7}{2} + C = 0 \Leftrightarrow C = -\dfrac{7}{2}$

Pour $x \in \left]-\infty ; -3\right[$: $F(x) = -\dfrac{2}{x+3} + \dfrac{3}{2(x+3)^2} - \dfrac{7}{2}$

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Sur $\mathbb{R}$, $f(x) = (2xe^{x})^{2}$ et $G(x) = (ax^{2} + bx + c)e^{2x}$

1) $(uv)' = u'v + uv'$ $(e^{u})' = u'e^{u}$

$G'(x) = (2ax + b) \times e^{2x} + (ax^{2} + bx + c) \times 2 \times e^{2x} = e^{2x}(2ax + b + 2ax^{2} + 2bx + 2c)$

$= \boxed{e^{2x}}\left(\boxed{2a}x^{2} + \boxed{(2a+2b)}x + \boxed{b+2c}\right)$

2) $G' = f$ avec $f(x) = \boxed{4}x^{2}\boxed{e^{2x}}$.

Par identification :

$\begin{cases} 2a = 4 \\ 2a + 2b = 0 \\ b + 2c = 0 \end{cases} \iff \begin{cases} a = 2 \\ b = -2 \\ c = 1 \end{cases}$

donc : $G(x) = (2x^{2} - 2x + 1)e^{2x}$

G est <u>une</u> primitive de f.