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dimanche 22 mars 2020 21:15

Rappel $\displaystyle\int_a^b f(t)\,dt = \Big[\underbrace{F(t)}_{\text{primitive de } f}\Big]_a^b = F(b) - F(a)$

Ex 99 p : 203

1) Signe de $x^2 - 4x + 3$? $\Delta = 4$ $x_1 = 1$ et $x_2 = 3$

x	$-\infty$		1		3		$+\infty$
Signe x^2-4x+3		+	0	$-$	0	+	

Signe de $a = 1$

2) $\displaystyle\int_1^5 |x^2-4x+3|\,dx = \underbrace{\int_1^3 |x^2-4x+3|\,dx}_{1 \leq x \leq 3} + \underbrace{\int_3^5 |x^2-4x+3|\,dx}_{3 \leq x \leq 5} = \int_1^3 -(x^2-4x+3)\,dx + \int_3^5 x^2-4x+3\,dx$

$= \left[-\frac{1}{3}x^3 + 4\times\frac{1}{2}x^2 - 3x\right]_1^3 + \left[\frac{1}{3}x^3 - 4\times\frac{1}{2}x^2 + 3x\right]_3^5 = \left(-\frac{1}{3}\times 3^3 + 2\times 3^2 - 3\times 3\right) - \left(-\frac{1}{3} + 2 - 3\right) + \frac{1}{3}\times 5^3 - 2\times 5^2 + 3\times 5\Big)$

$= \left(-\frac{1}{3}\times 3^3 + 2\times 3^2 - 3\times 3\right) - \left(-\frac{1}{3} + 2 - 3\right) + \left(\frac{1}{3}\times 5^3 - 2\times 5^2 + 3\times 5\right) - \left(\frac{1}{3}\times 3^3 - 2\times 3^2 + 3\times 3\right) = 0 + \frac{4}{3} + \frac{20}{3} - 0 = 8$

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1) $\displaystyle\int_0^{\pi/2} \frac{\cos x}{(1+\sin x)^{3}}\,dx$

$\displaystyle\int \frac{u'}{u^n} = -\frac{1}{n-1} \times \frac{1}{u^{n-1}}$ si $n \neq 1$

$u(x) = 1 + \sin x$ donc $u'(x) = \cos x$

$$\int_0^{\pi/2} \frac{\cos x}{(1+\sin x)^3}\,dx = \left[-\frac{1}{2} \times \frac{1}{(1+\sin x)^2}\right]_0^{\pi/2} = \left(-\frac{1}{2} \times \frac{1}{(1+\sin \frac{\pi}{2})^2}\right) - \left(-\frac{1}{2} \times \frac{1}{(1+\sin 0)^2}\right) = -\frac{1}{8} - \left(-\frac{1}{2}\right) = \frac{3}{8}$$

$= 0,875$

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2) $\int_{-\pi/4}^{\pi/4} (\tan x)^2 dx$

$$(\tan x)' = \left(\frac{\sin x}{\cos x}\right)' = \frac{\cos x \times \cos x - \sin x \times (-\sin x)}{(\cos x)^2} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$$

$$= \frac{1}{\cos^2 x} = \frac{\cos^2 x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x} = 1 + \tan^2 x$$

donc $(\tan x - x)' = 1 + \tan^2 x - 1 = \tan^2 x$

$$\int_{-\pi/4}^{\pi/4} (\tan x)^2 dx = \left[\tan x - x\right]_{-\pi/4}^{\pi/4} = \left(\tan\frac{\pi}{4} - \frac{\pi}{4}\right) - \left(\tan\left(-\frac{\pi}{4}\right) - \left(-\frac{\pi}{4}\right)\right) = 1 - \frac{\pi}{4} - \left(-1 + \frac{\pi}{4}\right)$$

$$= 2 - \frac{\pi}{2} \simeq 0,429$$

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5) $\displaystyle\int_0^{\pi/4} \frac{\cos 2x}{(2+3\sin 2x)^3}\,dx$

A faire…….