Ex 57 p : 200

1) $f(x) = \boxed{(3x-1)}\underbrace{(3x^2-2x+3)}_{u}^{3}$

$\int u'u^n = \frac{1}{n+1} u^{n+1}$ avec $n \neq -1$

$u(x) = 3x^2 - 2x + 3$

$u'(x) = 6x - 2$

$f(x) = \boxed{\frac{1}{2} \times (6x-2)} \times (3x^2-2x+3)$

$F(x) = \frac{1}{8}(3x^2-2x+3)^4 + c$ avec c constante réelle.

2) $f(x) = (x^3-3x+4)^5(2x^2-2)$

$u(x) = x^3 - 3x + 4$

$u'(x) = 3x^2 - 3$

$3x^2 - 3 \xrightarrow{\times \frac{2}{3}} 2x^2 - 2$

$f(x) = \frac{2}{3}(3x^2-3) \times (x^3-3x+4)^5$

$F(x) = \frac{1}{9}(x^3-3x+4)^6 + c$

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3) $f(x) = 4(\underbrace{4x+3}_{u})^3$ donc $F(x) = \frac{1}{4}(4x+3)^4 + c$.

u'

4) $f(x) = (-2x+1)^4$.

$u(x) = -2x+1$
$u'(x) = -2$

donc $f(x) = -\frac{1}{2} \times -2 \times (-2x+1)^4$ donc $F(x) = -\frac{1}{10}(-2x+1)^5 + c$

Ex 63 p. 200

1) $f(x) = \frac{7}{(x-1)^4}$

$\int \frac{u'}{u^n} = -\frac{1}{n-1} \times \frac{1}{u^{n-1}}$ avec $n \neq 1$

$u(x) = x-1$
$u'(x) = 1$

$f(x) = 7 \times \frac{1}{(x-1)^4}$

$F(x) = 7 \times \left(-\frac{1}{3}\right) \times \frac{1}{(x-1)^3} + c = -\frac{7}{3} \times \frac{1}{(x-1)^3} + c$

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2) $f(x) = \dfrac{9}{(4x+1)^3}$

$u(x) = 4x+1$
$u'(x) = 4.$

$f(x) = \dfrac{9}{4} \times \dfrac{4}{(4x+1)^3}$ donc $F(x) = \dfrac{9}{4} \times \left(\dfrac{-1}{2}\right) \times \dfrac{1}{(4x+1)^2} + c$

$F(x) = -\dfrac{9}{8} \times \dfrac{1}{(4x+1)^2} + c$

3) $f(x) = \dfrac{2 \times 1}{3(2x+3)^2}$

$\dfrac{a}{b} \times \dfrac{c}{d} = \dfrac{ac}{bd}$

$u(x) = 2x+3$
$u'(x) = 2$

donc $f(x) = \dfrac{1}{3} \times \dfrac{2}{(2x+3)^2}$ donc $F(x) = \dfrac{1}{3} \times (-1) \times \dfrac{1}{(2x+3)} + c$

$F(x) = -\dfrac{1}{3} \times \dfrac{1}{(2x+3)} + c$

Page 4 : ex 69 page 201

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1) $f(x) = \frac{5}{\sqrt{2x+1}}$

$\int \frac{u'}{\sqrt{u}} = 2\sqrt{u}$

$u(x) = 2x+1$

$u'(x) = 2$.

$f(x) = \frac{5}{2} \times \frac{2}{\sqrt{2x+1}}$ donc $F(x) = \frac{5}{2} \times 2\sqrt{2x+1} + c = 5\sqrt{2x+1} + c$

2) $f(x) = \frac{-2}{\sqrt{x-6}}$

$u(x) = x-6$

$u'(x) = 1$.

$f'(x) = -2 \times \frac{1}{\sqrt{x-6}}$ donc $F(x) = -4\sqrt{x-6} + c$

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3) $f(x) = \dfrac{1x - 1}{\sqrt{x(x-2)}}$

$u(x) = x(x-2) = x^2 - 2x$ donc $f(x) = \dfrac{1}{2} \times \dfrac{2x-2}{\sqrt{x(x-2)}}$ donc $F(x) = \sqrt{x(x-2)} + c$

$u'(x) = 2x - 2$

Page 6: démonstration de la propriété 4 : intégrale d'une fonction continue, positive.

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Dém : (H) f fonction continue, positive sur $[a; b]$

F une primitive de f.

cf $\left(\int_a^x f(t)\,dt\right)' = f(x)$

cours chap. 1

$G(x) = \int_a^{x} f(t)\,dt$ donc G est une primitive de f

il existe une constante k tq pour tout x de $[a; b]$, $F(x) = G(x) + k$

$\left(\begin{array}{l} G'(x) = f(x) \\ F'(x) = f(x) \end{array}\right)$ or $\int_a^b f(t)\,dt = G(b) = F(b) - k$

$G(a) = \int_a^a f(t)\,dt = 0$ $F(a) = G(a) + k = 0 + k$

donc $\int_a^b f(t)\,dt = F(b) - k = F(b) - F(a)$

Page 7 : exemple

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$$\int_0^{\pi/2} \underbrace{\boxed{\cos t}}_{f}\, dt = \Big[\underbrace{\boxed{\sin t}}_{F} \Big]_0^{\pi/2} = \sin\left(\frac{\pi}{2}\right) - \sin 0 = 1 - 0 = 1$$

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4) $\int_{-1}^{0} \underbrace{e^{-2x+1}} dx$

$f(x) = e^{-2x+1}$. Primitive de e^{-2x+1} ?

forme $\int u'e^u = e^u$

$u(x) = -2x+1$ $\qquad f(x) = -\frac{1}{2} \times \underbrace{(-2)}_{u'} \times e^{-2x+1}$. donc $F(x) = -\frac{1}{2} e^{-2x+1}$

$u'(x) = -2$

$$\int_{-1}^{0} e^{-2x+1} dx = \left[-\frac{1}{2} e^{-2x+1} \right]_{-1}^{0} = -\frac{1}{2} e^{-2\times 0+1} - \left(-\frac{1}{2} e^{-2\times(-1)+1} \right)$$

$$= -\frac{1}{2} e + \frac{1}{2} e^3 \approx 8,68$$

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⚠ $e^a \times e^b = e^{a+b}$

1) $\int_{-1}^{1} e^x + e^{-x}\,dx$

Primitive de $f(x) = e^x + e^{-x}$

$F(x) = e^x - e^{-x}$

$u(x) = -x$

$u'(x) = -1$

$e^{-x} = -\underbrace{(-1)}_{u'} \times \underbrace{e^{-x}}_{e^u}$ donc primitive de e^{-x} est $-e^{-x}$

$$\int_{-1}^{1} e^x + e^{-x}\,dx = \left[e^x - e^{-x}\right]_{-1}^{1} = (e^1 - e^{-1}) - (e^{-1} - e^1) = e^1 - e^{-1} - e^{-1} + e^1 = 2e^1 - 2e^{-1}$$

$$= \boxed{2\left(e - \frac{1}{e}\right)}$$

Page 10 : suite ex 95

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2) $\int_0^1 \frac{1}{(2x+1)^2}\,dx$ -

$\int \frac{u'}{u^n} = -\frac{1}{n-1} \times \frac{1}{u^{n-1}}$ si $n \neq 1$

$f(x) = \frac{1}{(2x+1)^2}$

$u(x) = 2x+1$

$u'(x) = 2$.

$f(x) = \frac{1}{2} \times \frac{2}{(2x+1)^2}$ donc $F(x) = \frac{1}{2} \times (-1) \times \frac{1}{2x+1}$

$$\int_0^1 \frac{1}{(2x+1)^2}\,dx = \left[-\frac{1}{2} \times \frac{1}{2x+1}\right]_0^1 = -\frac{1}{2} \times \frac{1}{2\times 1+1} - \left(-\frac{1}{2} \times \frac{1}{2\times 0+1}\right) = -\frac{1}{6} + \frac{1}{2} = \frac{1}{3}$$

3) $\int_0^1 (2x+1)^2\,dx$

$f(x) = (2x+1)^2$

$u(x) = 2x+1$

$u'(x) = 2$

$f(x) = \frac{1}{2} \times 2 \times (2x+1)^2$

F(x)

$$\int_0^1 (2x+1)^2\,dx = \left[\frac{1}{6}(2x+1)^3\right]_0^1 = \frac{1}{6} \times (3^3) - \frac{1}{6} \times 1^3 = \frac{27}{6} - \frac{1}{6} = \frac{13}{3}$$

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1) $\int_{-1}^{1/2} -2t+1 \, dt = \left[-t^2+t\right]_{-1}^{1/2} = -\left(\frac{1}{2}\right)^2+\frac{1}{2} - \left(-(-1)^2+(-1)\right) = \frac{1}{4} - (-2) = \frac{9}{4}$.

$\int_{1/2}^{3} 2t-1 \, dt = \left[t^2-t\right]_{1/2}^{3} = 3^2-3 - \left(\left(\frac{1}{2}\right)^2-\frac{1}{2}\right) = 6-\left(-\frac{1}{4}\right) = \frac{25}{4}$.

2) $|2t-1| = +2t-1$ si $t \geqslant 1/2$ $\qquad 2t-1 \geqslant 0 \iff 2t \geqslant 1 \iff t \geqslant \frac{1}{2}$.

$-(2t-1) = -2t+1$ sinon

$\int_{-1}^{3} |2t-1| \, dt = \int_{-1}^{1/2} |2t-1| \, dt + \int_{1/2}^{3} |2t-1| \, dt = \int_{-1}^{1/2} -2t+1 \, dt + \int_{1/2}^{3} 2t-1 \, dt = \frac{9}{4}+\frac{25}{4} = \frac{34}{4}$

$= \frac{17}{2}$.