Exponentielle complexe.

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Exercice 153 page 268

153. On considère les nombres complexes :
$z_1 = \sqrt{6} - i\sqrt{2}$ et $z_2 = 2 - 2i$.

1. Écrire sous forme algébrique le nombre complexe $\frac{z_1}{z_2}$.
2. Écrire sous forme exponentielle les nombres complexes z_1 ; z_2 et $\frac{z_1}{z_2}$.
3. En déduire les valeurs exactes de $\cos\frac{\pi}{12}$ et $\sin\frac{\pi}{12}$.

1) $\frac{z_1}{z_2} = \frac{\sqrt{6} - i\sqrt{2}}{2-2i} \times \frac{2+2i}{2+2i} = \frac{2\sqrt{6} + 2\sqrt{6}i - i\times\sqrt{2}\times 2 - i^2 2\sqrt{2}}{2^2 + 2^2}$

Rappel : $z = a + ib$
$z\bar{z} = |z|^2 = a^2 + b^2$

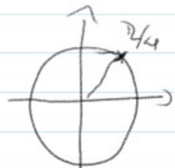
$\frac{z_1}{z_2} = \frac{2\sqrt{6} + 2\sqrt{2} + i(2\sqrt{6} - 2\sqrt{2})}{8} = \frac{\sqrt{6} + \sqrt{2}}{4} + i\frac{\sqrt{6} - \sqrt{2}}{4}$

2) $z_1 = \sqrt{6} - i\sqrt{2}$ $\quad |z_1| = \sqrt{(\sqrt{6})^2 + (-\sqrt{2})^2} = \sqrt{8}$

$\theta = \arg z_1 \pmod{2\pi}$ donc $\cos\theta = \frac{\sqrt{6}}{\sqrt{8}} = \frac{\sqrt{3}\times\sqrt{2}}{\sqrt{4}\times\sqrt{2}} = \frac{\sqrt{3}}{2} \iff \theta = \frac{\pi}{6} \pmod{2\pi}$ ou $-\frac{\pi}{6} \pmod{2\pi}$

$\sin\theta = -\frac{\sqrt{2}}{\sqrt{8}} = -\frac{1}{2}$ donc $\theta < 0$ mes. p^ale

ccl $\theta = -\frac{\pi}{6}$ $\quad z_1 = \sqrt{8}\,e^{-i\pi/6}$



$z_2 = 2 - 2i$ $\quad |z_2| = \sqrt{2^2 + (-2)^2} = \sqrt{8}$

$\theta' = \arg z_2 \pmod{2\pi}$ donc $\cos\theta' = \frac{2}{\sqrt{8}} = \frac{2}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2} \iff \theta' = \frac{\pi}{4} \pmod{2\pi}$ ou $\theta' = -\frac{\pi}{4} \pmod{2\pi}$

$\sin\theta' = -\frac{2}{\sqrt{8}} = -\frac{\sqrt{2}}{2} < 0$ donc $\theta' < 0$ mes. p^ale

$\theta' = -\frac{\pi}{4} \pmod{2\pi}$ $\quad z_2 = \sqrt{8}\,e^{-i\pi/4}$

$\frac{z_1}{z_2} = \frac{\cancel{\sqrt{8}}\,e^{-i\pi/6}}{\cancel{\sqrt{8}}\,e^{-i\pi/4}} = e^{-i\pi/6 - (-i\pi/4)} = e^{i(-\pi/6 + \pi/4)} = e^{i(-\frac{2\pi}{12} + \frac{3\pi}{12})} = e^{i\pi/12}$

3) $e^{i\pi/12} = \cos\frac{\pi}{12} + i\sin\frac{\pi}{12} = \frac{\sqrt{6} + \sqrt{2}}{4} + i\frac{\sqrt{6} - \sqrt{2}}{4}$

2 nb complexes sont égaux ssi ils ont m̂ partie réelle et m̂ partie imaginaires

$$\begin{cases} \cos\pi/12 = \frac{\sqrt{6} + \sqrt{2}}{4} \\ \sin\pi/12 = \frac{\sqrt{6} - \sqrt{2}}{4} \end{cases}$$

Exponentielle complexe

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Exercice 159 page 269

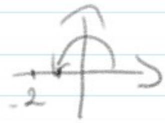
159. On donne $z_1 = -2e^{i\frac{\pi}{2}}$, $z_2 = ie^{i\frac{\pi}{3}}$ et $z_3 = 4e^{-i\frac{2\pi}{3}}$.

1. Donner la forme exponentielle de z_1 et z_2.
2. Donner la forme exponentielle de $z_1z_2z_3$ et $z_2(z_1)^2$.
3. Donner la forme exponentielle de $(z_1)^3$.

1) $z_1 = -2e^{\frac{i\pi}{2}} = 2e^{i\pi} \times e^{i\pi/2} = 2e^{i\left(\pi + \frac{\pi}{2}\right)}$

$= 2e^{3i\pi/2}$

$-2 = 2e^{i\pi}$



$i = 1e^{i\pi/2}$

$z_2 = e^{i\pi/2} \times e^{i\pi/3} = e^{i(\pi/2 + \pi/3)} = e^{5i\pi/6}$

2) $z_1z_2z_3 = 2e^{3i\pi/2} \times e^{5i\pi/6} \times 4e^{-i\times 2\pi/3} = 8e^{i(3\pi/2 + 5\pi/6 - 2\pi/3)} = 8e^{5i\pi/3}$

$z_2 \times (z_1)^2 = e^{5i\pi/6} \times \left(2e^{3i\pi/2}\right)^2 = 4e^{i(5\pi/6 + 2\times 3\pi/2)} = 4e^{23i\pi/6} = 4e^{(24i\pi/6 - \pi i/6)} = 4e^{-i\pi/6}$

$z_1^3 = \left(2e^{3i\pi/2}\right)^3 = 2^3 \times \left(e^{3i\pi/2}\right)^3 = 8 \times e^{9i\pi/2} = 8e^{i\left(\frac{8\pi}{2} + \frac{\pi}{2}\right)} = 8e^{i\pi/2}$