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ex : 55 p : 200

1) $f(x) = \boxed{2x}\,\boxed{(x^2+3)}^4$

u' $\quad$ u

$(u^n)' = n\,u'\,u^{n-1}$

$u(x) = x^2 + 3$

$u'(x) = 2x$

$F(x) = \frac{1}{5}(x^2+3)^5 + k$

$\int u'u^n = \frac{1}{n+1}u^{n+1}$ car $\left(\frac{1}{n+1}u^{n+1}\right)' = \frac{1}{\cancel{n+1}} \times \cancel{(n+1)}\,u'u^n$

si $n \neq -1$

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$$\int u' u^n = \frac{1}{n+1} u^{n+1}$$

si $n \neq -1$

$$f(x) = \underbrace{(3x^2 - 2)}_{u'} \times \underbrace{(x^3 - 2x + 3)}_{u^n}{}^{3}$$

$n = 3$ $\quad u(x) = x^3 - 2x + 3$

$u'(x) = 3x^2 - 2$

$F(x) = \frac{1}{4}(x^3 - 2x + 3)^4 + c$ avec c constante

3) $f(x) = \underbrace{(2x+1)}_{u'}\underbrace{\boxed{(x^2 + x + 1)}}_{u}{}^{5}$

$n = 5$ $\quad u(x) = x^2 + x + 1$

$u'(x) = 2x + 1$

$F(x) = \frac{1}{6}(x^2 + x + 1)^6 + c$ avec c constante

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$$\int u' u^n = \frac{1}{n+1} u^{n+1}$$

si $n \neq -1$

Ex 56 p: 200

$f(x) = \sin x \times \boxed{(\cos x)}^4$

$n = 4$ $\quad u(x) = \cos x$

$u'(x) = -\sin x$

$f(x) = - \underbrace{(-\sin x)}_{u'} \times \underbrace{(\cos x)^4}_{u^4}$

$F(x) = - \times \frac{1}{5} \times (\cos x)^5 = -\frac{1}{5}(\cos x)^5$ donc toutes les primitives de f sont de la forme $F(x) = -\frac{1}{5}(\cos x)^5 + k$ avec k constante réelle.

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$$\int u' u^n = \frac{1}{n+1} u^{n+1}$$

si $n \neq -1$

2) $f(x) = \boxed{(x+1)}\boxed{(x^2+2x-5)}^4$

$n=4$

$u(x) = \overbrace{x^2+2x-5}^{u}$

$u'(x) = 2x+2$

$f(x) = \boxed{\frac{1}{2}\underbrace{(2x+2)}_{u'}} \times (x^2+2x-5)^4$ car $\frac{1}{2}(2x+2) = x+1$

$F(x) = \frac{1}{2} \times \frac{1}{5} \times (x^2+2x-5)^5 + C = \frac{1}{10}(x^2+2x-5)^5$

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$$\int u'u^n = \frac{1}{n+1}u^{n+1}$$

$n \neq -1$

3) $f(x) = (2-x)\boxed{(x^2-4x+2)}^5$

u

$n = 5$ $\quad u(x) = x^2-4x+2$

$u'(x) = 2x-4$

$$f(x) = -\frac{1}{2} \times (2x-4) \times (x^2-4x+2)^5$$

$$F(x) = -\frac{1}{2} \times \frac{1}{6}(x^2-4x+2)^6 + C = -\frac{1}{12}(x^2-4x+2)^6 + C$$

$2x-4 = 2(x-2) = (-2)\times(x-2)$

$$x-2 \xrightarrow{\times(-2)} 2x-4$$

$$2x-4 \xrightarrow{\times\left(-\frac{1}{2}\right)} x-2$$

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Ex 61 p : 200

$\sin^3 x = (\sin x)^3 = \sin x \times (\sin x)^2$

$= \sin x \times (1 - \cos^2 x)$

$= \sin x - \sin x \cos^2 x$

or $\begin{cases} \cos^2 x + \sin^2 x = 1 \\ \text{donc } \sin^2 x = 1 - \cos^2 x \end{cases}$

$(u+v)' = u' + v'$

donc $\int u + v = \int u + \int v$

$\int \sin x = -\cos x$

$\int u' u^n = \frac{1}{n+1} u^{n+1}$

si $n \neq -1$

On pose $n = 2$ $\quad u(x) = \cos x$

$u'(x) = -\sin x$

$\underbrace{-\sin x}_{u'} \times \underbrace{(\cos x)^2}_{u^2}$

$F(x) = \boxed{-\cos x} + \frac{1}{3} \times (\cos x)^3 + k$

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1) $f(x) = \dfrac{2x+5}{(x^2+5x)^4}$

$$\int \frac{u'}{u^n} = -\frac{1}{n-1} \times \frac{1}{u^{n-1}}$$

si $n \neq 1$

Cours : $\int u' u^n = \dfrac{1}{n+1} \times u^{n+1}$

or $\dfrac{u'}{u^n} = u' u^{-n}$

donc $\int \dfrac{u'}{u^n} = \int u' u^{-n} = \dfrac{1}{-n+1} \times u^{-n+1} = \dfrac{1}{-(n-1)} \times \dfrac{1}{u^{n-1}}$ car $-n+1 = -(n-1)$

$= -\dfrac{1}{n-1} \times \dfrac{1}{u^{n-1}}$

$n = 4$ $u(x) = x^2 + 5x$

$u'(x) = 2x + 5$

donc $f(x) = \dfrac{u'}{u^4}$ donc $F(x) = -\dfrac{1}{3} \times \dfrac{1}{(x^2+5x)^3} + k$

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2) $f(x) = \dfrac{3x^2+2}{(x^3+2x)^2}$

$$\int \frac{u'}{u^n} = -\frac{1}{n-1} \times \frac{1}{u^{n-1}} \quad \text{si } n \neq 1$$

$n = 2$ $\quad u(x) = x^3 + 2x$

$\quad u'(x) = 3x^2 + 2$

donc $F(x) = -\dfrac{1}{1} \times \dfrac{1}{(x^3+2x)} + C = -\dfrac{1}{x^3+2x} + C$

3) $f(x) = \dfrac{x}{(x^2+1)^3}$

$$\int \frac{u'}{u^n} = -\frac{1}{n-1} \times \frac{1}{u^{n-1}} \quad \text{si } n \neq 1$$

$n = 3$ $\quad u(x) = x^2 + 1$

$\quad u'(x) = 2x$

$f(x) = \dfrac{1}{2} \times \dfrac{2x}{(x^2+1)^3}$ donc $F(x) = \dfrac{1}{2} \times \left(-\dfrac{1}{2}\right) \times \dfrac{1}{(x^2+1)^2} + C$

$F(x) = -\dfrac{1}{4} \times \dfrac{1}{(x^2+1)^2} + C$

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ex: 68 p. 201

1) $f(x) = \dfrac{3}{\sqrt{3x+1}}$

$$(\sqrt{u})' = \frac{u'}{2\sqrt{u}} \quad \times 2$$

$$(2\sqrt{u})' = \frac{u'}{\sqrt{u}}$$

donc $\displaystyle\int \frac{u'}{\sqrt{u}} = 2\sqrt{u}$

$u(x) = 3x+1$

$u'(x) = 3$

donc $F(x) = 2\sqrt{3x+1} + C$

$\displaystyle\int \frac{u'}{\sqrt{u}} = 2\sqrt{u}$

2) $f(x) = \dfrac{1}{\sqrt{4-5x}}$

$u(x) = 4-5x$

$u'(x) = -5$

donc $f(x) = -\frac{1}{5} \times \dfrac{-5}{\sqrt{4-5x}}$ donc $F(x) = -\frac{1}{5} \times 2\sqrt{4-5x} + c$

$= -\frac{2}{5}\sqrt{4-5x} + c$

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ex 68 p: 201

3) $f(x) = \dfrac{x}{\sqrt{x^2+1}}$

$\int \dfrac{u'}{\sqrt{u}} = 2\sqrt{u}$

$u(x) = x^2+1$

$u'(x) = 2x$

$f(x) = \dfrac{1}{2} \times \dfrac{2x}{\sqrt{x^2+1}}$ donc $F(x) = \dfrac{1}{2} \times 2\sqrt{x^2+1} + C$

$= \sqrt{x^2+1} + C$

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Ex 72 p: 201

1) $f(x) = \cos(2x)$

$u(x) = 2x$

$u'(x) = 2$

$\int u' \cos u = \sin u$

donc $f(x) = \frac{1}{2} 2 \times \cos(2x)$ donc $F(x) = \frac{1}{2} \times \sin(2x) + c$ avec c c^{ste}

2) $f(x) = e^{-x}$

donc $u(x) = -x$

$u'(x) = -1$

$\int u' \exp u = \exp u$ car $(\exp u)' = u' \times \exp u$

$f(x) = -(-1) \times e^{-x}$ donc $F(x) = -e^{-x} + C$

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$$\int u'u^n = \frac{1}{n+1} \times u^{n+1} \quad \text{si } n \neq -1$$

3) $f(x) = x^3 (x^4 - 2)^3$

$u(x) = x^4 - 2$

$u'(x) = 4x^3$

$n = 3$

donc $f(x) = \frac{1}{4} \times \underbrace{4x^3}_{u'} \times \underbrace{(x^4-2)^3}_{u^n}$

donc $F(x) = \frac{1}{4} \times \frac{1}{4} \times (x^4 - 2)^{\cancel{3}4} + C = \frac{1}{16}(x^4 - 2)^{\cancel{3}4} + C$